A comparison of methods used in measuring finite strains from ellipsoidal objects

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(Received 20 August 1982; accepted in revised form 10 March 1983)

Abstract—The available methods for measuring strains in two and three dimensions from passive ellipsoidal objects are compared in an attempt to determine the most useful and precise procedure for the structural geologist. A comparison of data collection techniques showed that the use of thin orthogonally cut slabs or mylar overlays used with a monocomparater provides the most reproducible data. Lisle's theta-curve and Shimamoto and Ikeda's algebraic methods provided the most precise, and probably most accurate, two-dimensional data while Miller and Oertel's procedure and possibly Dunnet's PHASE 5 program gave the best three-dimensional results.

An examination of errors encountered in strain analyses suggests that all of the available methods give accurate orientations of the finite strain ellipsoid. However, the magnitudes of strain ratios show large variations that are dependent on the sample size and procedure used. Shimamoto and Ikeda's method again proved to be the most precise, giving reproducible results with as few as ten elliptical objects.

The samples used in the above comparison are part of a larger analysis of strains occurring in southeastern Maine. Structural elements observed in four selected areas of Avalonian belt rocks and the strain data collected suggest that the region has undergone at least three non-coaxial deformations with $D_1 > D_2 > D_3$.

INTRODUCTION

FINITE strain analyses provide an important and often overlooked tool for unraveling the magnitudes and orientations of strains and the relative importance of deformation mechanisms in a variety of deformed terrains. Without strain data it has proved impossible to apportion the total finite strain between separate deformational pulses in a polydeformed region. Such data are also being used to determine the tectonic thinning of units (Tobisch *et al.* 1977), to look at the relative mechanical behavior of adjacent lithologic units (Paterson 1981a), and to unfold deformed structures (Oertel 1974).

A variety of techniques have recently been published that allow the calculation of finite strains from passive spherical and ellipsoidal objects. Hanna & Fry (1979) have contrasted several of the techniques using data from ooliths in a deformed limestone. The present study is a comparative analysis of techniques of collecting data for, and measuring strains from, ellipsoidal objects and, in particular conglomerates.

Previous work

Ramsay (1967) introduced or discussed many of the techniques that form the foundation of present-day strain analyses. He examined problems encountered in different methods due to initial particle shapes and presented methods for combining two-dimensional data in order to obtain axial ratios in three dimensions.

Dunnet (1969) and Dunnet & Siddans (1971) presented a refined procedure for using $\log R_f/\phi$ plots for the determination of finite strains. Dunnet also noted that the final axial ratio (R_f) and orientation (ϕ) of an elliptical object subjected to a strain of magnitude (R_s) and orientation (β) depend on the initial particle shape (R_i), orientation (θ), and the ductility contrast between the particles and surrounding matrix.

Elliott (1970) noted several apparent advantages of plotting log R_t/ϕ data on a polar plot. He suggested that the major advantages of using polar plots are that they bring out the inherent symmetry in R_t/ϕ plots, allow for the rapid manipulation of graphical data using the Shape Factor grid, and provide ease in recognizing undeformed particle populations.

Matthews *et al.* (1974) and Shimamoto & Ikeda (1976) developed algebraic methods for measuring strains from random ellipsoidal objects. Both of these methods proved useful in the analysis of conglomerates.

Lisle (1977, 1979) examined the values obtained and errors encountered in using the arithmetic, geometric and harmonic means (Table 1). He concluded that the harmonic mean gave the most accurate results (Fig. 1), although it still represented an overestimate of strain. Lisle also presented his 'theta-curve' method which calculates finite strains assuming an initial random distribution of particles.

Hanna & Fry (1979) used the method of Ramsay (1967), Dunnet & Siddans (1971), Shimamoto & Ikeda (1976), Mimran (1976), and Fry (1979) to measure different spherical objects and compare the results. It is interesting to note that with the exception of Mimran's technique, the procedures gave what appear to be reasonably consistent values (Table 2).

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Table 1. Definitions of symbols used in this paper

Symbol	Definition
x_1, x_2, x_3	orthogonal reference axes
$x \ge y \ge z$	principal axes of ellipsoidal particle
$X \ge Y \ge Z$	principal axes of strain ellipsoid
<i>R</i> .	axial ratio, undeformed particle or ellipse
R.	axial ratio, deformed particle or ellipse
R.	axial ratio, strain ellipse
θ	angle from undeformed ellipse long axis to x_1
φ	angle from x to x_1
B	angle from direction maximum extension to x_1
r E	extension or elongation
Ē	log or natural strain
λ	quadratic elongation
N	number of particles or ellipses
f	fluctuation
Ŕ	arithmetic mean
Ĝ	geometric mean
Ŭ Ħ	harmonic mean

Table 2. Data presented in Hanna & Fry (1979). Note the similarity in values obtained. Methods used described in Hanna & Fry (1979)

Sample plane:	1	2	3
Method:	1 74	2.52	7 71
A: Ramsay long/short	1.74	2.52	3.31
B: Centre-to-centre	1.78	2.44	3.50
C: Dunnet & Siddans R_f/ϕ	1.65	2.40	3.25
D: Fry (graphical)	1.66	2.42	3.27
E: Shimamoto & Ikeda	1.68	2.41	3.26
F: Dunnet & Siddans (numerical)	1.69	2.42	3.24
G: Mimran	1.23	1.42	1.76

Seymour & Boulter (1979) analysed the errors encountered if incorrect assumptions were made about initial particle distributions. Errors as great as 94% were encountered using the methods of Dunnet & Siddans (1971) and Matthews *et al.* (1974).

Siddans (1980) combined different two-dimensional



Fig. 1. The relationship between arithmetic, geometric, and harmonic means and tectonic strains obtained by Lisle (1977).

data into three-dimensional results. The accuracy of the three-dimensional results were a function of the twodimensional method used, with Dunnet's R_f/ϕ technique giving the best results. Siddans also noted that all methods gave a slight overestimate of the true value of strain determined by computer modelling of deformed ellipsoidal objects.

Geological interpretation of sample localities

Some of the most highly deformed rocks in the Eastern Maine Coastal belt, or Avalonian Platform, occur in the Camden–Rockland area 50 km south of Bangor, Maine (Fig. 2). To the west of this region lie the NE-trending



Fig. 2. Index map of southeastern Maine showing the towns and major tectonic features in the area. Adapted from Osberg & Guidotti (1974).

Liberty–Orrington antiform and Merrimack synclinorium. To the east lie Siluro–Devonian metavolcanic sequences, locally overlying older units.

The stratigraphy in the Camden area is broken into several fault-bounded sequences (Osberg & Guidotti 1974, Bickel 1976). The Penobscot Bay and Rockland sequences have been divided into the Benner Hill, Rockport, and Megunticook sequences, and the Jam Brook Formation (Osberg & Guidotti 1974).

The samples in the present study (Table 3) were obtained from quartzite, limestone and polymictic conglomerates from the Benner Hill, Rockport, and Megunticook sequences, and the Jam Brook Formation. The long axes of pebbles examined, ranged from 2 to 200 mm in length (granule to cobble gravels on the Wentworth scale). Because of the high percentage of clasts vs matrix, similarities in matrix and clast lithologies, and low values of ductility contrasts obtained using Gay's (1968) method, ductility contrasts between clasts and matrix materials were thought to be small in many of the samples.

The age of the rocks sampled remains uncertain because of the difficulty in correlating units between fault blocks (Osberg & Guidotti 1974). The Benner Hill sequence contains Caradocian brachiopods (Boucot *et al.* 1972), an age which is in good agreement with the whole rock date of 460 ± 10 Ma obtained by Brookins (1976). Brookins (1976) also obtained a date of 425 ± 35 Ma for the lowest member of the Megunticook sequence.

All four sampled regions have experienced at least three periods of deformation. An examination of strains and structural elements in each region suggests that the three main periods of deformation were usually non-coaxial with the intensity sequence being $D_1 > D_2 > D_3$ Table 3. The lithology, number, and location of samples used in the present study. Three faces were examined for each sample. Thus, 246 two-dimensional and 82 three-dimensional values of strain were compared in this study. The sequences and formations are discussed in the text

Lithology	Number of samples
Rockport Sequence:	
Quartz Conglomerates	25
Limestone Conglomerates	24
Polymictic Conglomerates	4
Mica Schists (with clasts)	2
Greywacke (with clasts)	1
Benner Hill Sequence:	
Quartz Biotite 'Grits'	3
Mica Schist (with clasts)	1
Megunticook Sequence:	
Quartz Conglomerates (Battie)	10
Jam Brook Formation:	
Quartz Biotite Conglomerates	12
Total samples	82

(Paterson 1981a). The structural elements observed and the tentative information on finite strains for the region are listed in Table 4. The interpretation of this data and a more complete discussion of the sample localities will be presented in a future paper.

COLLECTION OF DATA FROM CONGLOMERATES

Obtaining strain data from conglomerates requires 15–30 h per sample. Thus the procedure that allows the greatest amount and most accurate information to be obtained should be used. In the present study, oriented samples containing 10–100 deformed objects were obtained. A structural inventory that included the orientations of foliations, bedding and lineations, local vari-

Table 4. Structural elements observed and strain data measured for three to five postulated deformational episodes in Avalonian belt rocks, southeastern Maine

	Pre- or Early D_1	D_1	D_2	D_3	Post D_3
	rootless, isoclinal fold hinges (?)	isoclinal, reclined or recumbent folds	open to tight asymmetrical folds (F_2)	open, upright folds (F_3)	high angle faults
ral Elements	mineral layering parallel to S_0 bedding microlithons with schistosity at angle to S_0	well developed schistosity (S_1) pebble and mineral lineation (L_1)	solution or slip cleavage (S_2) L_1 folded	poorly developed spaced cleavage complex fold patterns	regional jointing
Structu		boudinage of S_0 extension gashes	superimposed fold patterns minor thrusting of	fracturing of pebbles	→ ?
		pressure shadows around clasts major thrust faults development of cata- clastic textures	<i>F</i> ₁ limbs solution rims on limestone clasts	→ ?	
	X/Z = 2.00	X/Z = 9.00	X/Z = 4.00	X/Z = 1.5	
¥.	Z perpendicular to S_0	X parallel and per- pendicular to F,	X parallel to F_2	X perpendicular to F_3	no information
itrain Dai		constrictional in limbs, flattening in hinge zone	flattening XY parallel to S_2	flattening	
		XY parallel to S_1	<i>XY</i> total finite strain parallel to <i>S</i> ₁		

ations in lithology, the position of the sample within larger structural elements, and evidence for the nonhomogeneity of strain mechanisms was completed before each sample was removed. A rapid survey of strains in surrounding exposures was completed and variations in strain taken into account during subsequent sampling.

Laboratory preparation of samples

Three orthogonal $(\pm 5^{\circ})$ planes were cut in each sample. The massive nature of most conglomerates discourages the practice of making the orthogonal cuts parallel or perpendicular to foliations, and no attempt was made to do so. Ramsay (1967) noted that it is not necessary to make cuts parallel to the principal planes of the strain ellipsoid, and Siddans (1980) has recently confirmed Ramsay's statement.

Each orthogonal cut was cleaned, labelled xy, yz, xz (Table 5) in a right-handed coordinate system, and coated with acrylic spray. The orientations of each orthogonal plane and axis were determined in the field and recorded.

Table 5. The lithologies, numbers and clast sample sizes of specimens discussed in this paper. Note, however, that the results presented here are based on the analysis of all the samples listed in Table 3

Sample	Nun mea	nber of isured i	clasts n cut	Formation or	Lithology of
No.	xy	yz	xz	sequence	clasts and matrix
2	30	18	30	Rockport	Quartzite
6	20	33	21	Rockport	Limestone
11	23	28	13	Rockport	Quartzite
80	64	23	45	Rockport	Limestone
121	27	27	39	Jam Brook	Polymictic
140	29	22	23	Rockport	Quartzite
145	25	20	21	Rockport	Quartzite
158	39	22	29	Mt. Battie	Quartzite
173	34	40	16	Perry	Polymictic

Data collection

The data needed to calculate strains from deformed particles are the axial ratio (R_f) and orientations (ϕ) of the ellipses that approximate to the particles. Methods for obtaining this data have been discussed by Cloos (1947), Ramsay (1967) and Dunnet (1969). Three of the most commonly used methods were compared with several new procedures in this study.

The comparison of data collection procedures was completed by taking two separate faces on five samples displaying increasing degrees of strain. Photographs and mylar drawings of each face were prepared. Values of R_f and ϕ were measured from the rock face, photographs and mylar drawings. The harmonic means and average ϕ values were calculated from this data and are compared in Table 6.

All three methods provided consistent values for orientation (ϕ). The harmonic means calculated from each method did not vary more than 10% (see section on errors), falling within a typical range of errors encountered in dealing with strain analyses. Data from the photographs gave the most inconsistent results due to

Table 6. Values of the harmonic means of R_f and the average value of ϕ measured in rock faces, mylar overlays and photographs for 10 different rock slabs

		R _f			φ	
Sample No.	rock	mylar	photo	rock	mylar	photo
80xv	4.46	4.53	4.91	4	5	2
80 <i>vz</i>	2.24	2.31	2.25	20	18	23
158xy	1.45	1.51	1.40	32	27	28
158yz	1.57	1.56	1.62	-1	-2	5
6xv	1.52	1.36	1.61	-5	-10	-6
6vz	2.17	2.27	2.30	22	19	19
11xv	1.89	2.20	2.78	52	43	43
11vz	2.20	2.18	2.31	10	9	9
121xy	4.74	5.05	4.96	3	5	5
121 <i>yz</i>	2.39	2.10	2.27	0	2	0

the difficulty in observing particle edges and problems encountered in dealing with photographic paper. Mylar overlays provided a reasonably precise permanent record of each face and simplified the use of the monocomparater discussed below.

The implementation of several additional procedures improved the precision of determining strain ratios. Particles were grouped by size and lithology and values of strain determined separately for each group. Folded particles and particles near larger competent clasts were examined but not used (Dunnet 1969, Moore 1981). Particles not elliptical in shape were measured by estimating the axes of an inscribed ellipse (Robin 1977).

Difficulty was encountered in distinguishing pebbles from vein quartz, cataclastic fragments, rootless fold hinges and boudins. Initially every particle was measured and the ratios used for calculating strain data. If uncertainties about the origin of a particle existed, the data for that particle were removed and the strain recalculated. The use of this 'extra' data did not affect the values of ϕ but did, occasionally, alter the strain ratios.

Monocomparater

The principal axes of an ellipse intersect the diameter of that ellipse at four points (Fig. 3). The coordinates of the four points measured from a fixed origin can be determined using a monocomparater for ellipses or particles on thin slabs of rock, mylar drawings or photo-



Fig. 3. The X and Y components of four coordinates used in defining an ellipse with a monocomparater. The X/Y axes are set up by feeding coordinates representing the origin into the monocomparater.

graphs. The coordinates can be fed directly into a computer filespace and R_f and ϕ calculated using equations (1)–(4):

$$L_1 = ((X_2 - X_1)^2 + (Y_2 - Y_1)^2)^{1/2}, \qquad (1)$$

$$L_2 = ((X_4 - X_3)^2 + (Y_3 - Y_4)^2)^{1/2}, \qquad (2)$$

$$R_{\rm f} = L_1/L_2, \tag{3}$$

$$\phi = \frac{(X_2 - X_1)}{((X_1 - X_2)^2 + (Y_1 - Y_2)^2)^{1/2}},$$
 (4)

where X_N and Y_N are the coordinates of the points measured along the X and Y axes, respectively. Slightly more complex equations must be used for other labelling schemes and orientations (when $\phi > 90^\circ$) of an ellipse. Initial data obtained from using a monocomparater suggest that the precision of finite strain measurements improved from 10 to 15% errors to around 4% errors. The use of the monocomparater greatly increases the precision in strain measurements and enhances the rapid manipulation of strain data with computers.

Two-dimensional strain calculations

Five sample faces displaying decreasing values of strain were used to compare eleven methods of calculating two-dimensional strain. The magnitudes of principal strains calculated with each method are presented in Table 7. The orientations of principal strains were also calculated by determining the average value of ϕ and the vector mean. Both methods gave reliable results with the vector mean giving better results with low values of strain.

The values of R_s in Table 7 should decrease linearly in each row if the various methods can successfully distinguish decreasing values of strain (the precision is high). The columns should represent equivalent values of strain (if the accuracy of each method is the same). The conclusions of Siddans (1980) suggest that the most conservative value in each column represents the most accurate estimate of clast strains (not whole rock).

The X/Y plots gave the most inconsistent results because of the non-spherical nature of the objects used and the dependence of strains on clast size (Paterson 1981b). The three means maintained the relationship $\bar{R} < \bar{G} < \bar{H}$ previously mentioned by Lisle (1977). Of the four methods relying on $R_{\rm f}/\phi$ plots, Lisle's thetacurve method gave the most conservative and consistent results. All of the algebraic methods gave satisfactory results with Shimamoto & Ikeda's giving a slightly more conservative estimate.

Other considerations proved important in determining the value of each procedure. The methods of Lisle (1977), Shimamoto & Ikeda (1976) and Matthews *et al.* (1974) provide important information on errors encountered during calculations. All of the graphical techniques and Matthews *et al.* (1974) give information about initial particle shapes and their distributions. All of the methods can be adopted for use with a computer.

The above methods calculate the finite strains for clasts in a matrix. Gay (1968) presented one of the few methods available that attempt to calculate the strain experienced by the whole rock. Although there are several inherent problems in Gay's method, his procedure offers an important avenue for further research.

Fry (1979) and Mukhopadhyay (1980) present methods of calculating strains that may provide better values of whole rock strain. However, the use of Fry's method proved unsuccessful when used with conglomerates, probably due to their initial anisotropic spacing. Further attempts to use these methods are in progress.

Three-dimensional strain calculations

Five methods for calculating three-dimensional strain ratios are presently available in the literature. Cloos (1947) presented a method for use with initially spherical particles assuming no volume change has occurred. Ramsay (1967) calculates the principal strains by relating equations for ellipses and ellipsoids and by using the invariants of the strain ellipsoid. Oertel (1970, 1978) and Miller & Oertel (1979) present a method based on the use of weighting factors calculated from two-dimensional errors to estimate a strain ellipsoid. Roberts & Siddans (1971) and Siddans (1980) present a program, PHASE 5, that uses Ramsay's (1967) techniques to

Table 7. The comparison of two-dimensional finite strain ratios calculated using various techniques. Only the first author is listed for each technique. The references for and discussion of each method are presented in the section entitled 'Previous studies'. Although only 5 samples are presented here, 246 samples were measured and gave identical results to those presented above

		Sa	mple num	ber	
Methods	80xy	145 <i>xy</i>	80 <i>xz</i>	173 <i>xy</i>	158 <i>xy</i>
Arithmetic mean	4.99	3.63	2.91	2.50	1.57
Geometric mean	4.65	3.53	2.78	2.35	1.51
Harmonic mean	4.46	3.44	2.62	2.21	1.45
X/Y plot (Ramsay 1967)	6.67	3.67	3.51	3.20	1.74
$R_{\rm f}/\phi$ plot (Ramsay 1967)	4.78	3.25	3.00	2.18	1.64
$\log R_{\rm f}/\phi$ plot (Dunnet 1969)	4.77	3.29	2.74	2.18	1.57
Theta-curve (Lisle 1977)	4.05	3.19	2.41	1.96	1.27
Polar plot (Elliott 1970)	4.47	3.47	2.51	1.78	* n.d.
Algebraic (Matthews 1974)	4.47	3.54	2.53	1.95	1.47
Algebraic (Shimamoto 1976)	4.24	3.34	2.47	1.82	1.45
Algebraic (Robin 1977)	4.21	3.33	2.48	1.82	1.43

* Not able to detect any strain.

Table 8. Comparison of the magnitude and orientation of principal strains using the techniques of Miller & Oertel (1978) and Ramsay (1967). Several corrections have been made in Miller & Oertel's (1978) program. Sample sizes and lithologies are given in Table 5

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magnitudes of principal strains			
		Sample No.	
Method	2	140a	140b
Miller	2.32:1:.36	3.19:1:.58	2.83:1:.60
Ramsay	2.83:1:.45	3.45:1:.57	2.73:1:.63

	M	liller	Ra	imsay
	XYPlane	X Axis	XYPlane	X Axis
3	N8E 54NW	NS 11N	N5E 56NW	N1E 11N
140a	N40E 80SE	N42E 15NE	N44E 78SE	N43E 17N
l 40b	N38E 81SE	N41E 14NE	N37E 81SE	N40E 12N

calculate all six independent ellipsoids from two-dimensional data and find the average of these ellipsoids. Bell (1979) has recently presented a factorization procedure for calculating the strain ellipsoid. Bell (1979) suggests that the method is most useful for determining finite strains produced in one period of deformation after compaction.

Bell's (1979) procedure and the program PHASE 5 were not available at the time of writing. However, similarities in computational methods suggest that PHASE 5 should give results similar to Miller & Oertel's (1979) method. The method of Cloos (1947), because of assumptions mentioned above, gave overestimates of strain and inconsistent results from one sample to the next. These methods will not be considered further in this paper.

The magnitudes and orientations of principal strains X, Y, Z were calculated for several samples using the methods of Miller & Oertel (1979) and Ramsay (1967) (Table 8). Ramsay's method gave slightly less consistent results from one sample to another and less accurate values for each sample. Miller & Oertel gave the most accurate values of strain. The calculated orientations of the principal strains were again very accurate for all the methods used.

Errors in strain analyses

The errors present in strain analyses are of two types: random errors affecting the precision of each method, and systematic errors affecting the accuracy.

Any invalid assumption in a strain calculation will decrease the accuracy of the results. Thus large errors may be introduced by incorrectly analysing such factors as initial clast size and orientations, strain gradients, grain size and mechanism of strain. A study by Mosher (1976) pointed out how different assumptions about the mechanism of strain may change the values obtained by as much as 500%. Seymour & Boulter (1979) produced errors up to 94% when initially imbricate pebble fabrics were assumed to be random or bedding symmetric. Incorrect assumptions about other non-random fabrics produced errors of 10–20% in their study.

Lisle (1977) used mathematical models to test the accuracy of strain calculations using various mathematical means. Figure 1 obtained from Lisle (1977) presents the overestimates in strains obtained by using the arithmetic, geometric and harmonic means. Siddans (1980) tested the accuracy of three-dimensional strain calculations using the Fortran program PHASE 5. Errors in the orientation of principal strains were never greater than 3°. Errors in the magnitudes of principal strains varied from 14 to 16% depending on the method of two-dimensional analysis used (Table 9; estimated from fig. 6 in Siddans 1980).

Table 9. Overestimates of strains obtained when using the arithmetic and harmonic means, or Shimamoto & Ikeda's (1976), Matthews *et al.*'s (1974), and Dunnet's (1969) methods. Values estimated from fig. 6 in Siddans (1980)

Method	Percentage overestimate
Arithmetic mean	21.6
Harmonic mean	12.6
Shimamoto & Ikeda (1976)	12.4
Matthews et al. (1974)	9.4
Dunnet (1969)	6.6

The precision of each strain analysis is a function of the sample size (N), initial particle shape (R_i) , particle orientations (θ, ϕ) , and the magnitude of strain (Dunnet 1969, Matthews *et al.* 1974, Shimamoto & Ikeda 1976).

Matthews et al. (1974) derived an equation:

$$N_{\rm min} = 4\nu(\gamma_{\rm i})/(\epsilon R_{\rm s})^2, \qquad (5)$$

where (γ_i) is the variance of the algebraic components subscript i for an ellipse. The equation relates the minimum number of markers (N_{\min}) needed to obtain a certain percentage error (ϵR_s) at the 95% confidence limit. Values calculated from this equation are listed in Table 10 (Matthews *et al.* 1974).

 Table 10. Data presented in Matthews et al. (1974) relating the sample size to percentage error obtained in strain analyses

Number	Percentage error
13	20
24	15
53	10
212	5
5308	1

The minimum number of objects needed for each method depends on the type of object and method used. Dunnet (1969) suggested a minimum of 40 ooliths or pisoliths, and 60–100 pebbles for his technique. Matthews *et al.* (1974) noted that their conclusions are only strictly valid with a sample size of 20 or more. Shimamoto & Ikeda (1976) mentioned that their method may be accurate with as few as 10 ooliths. Tobisch *et al.* (1977), in using Elliott's (1970) polar plots, suggested that 42–70 markers are needed, depending on the type of marker. Other workers have used from 3 to 100 markers with various techniques (Cloos 1947, Brace 1955, Flinn 1956, Mukhopadhyay 1973).

Shimamoto & Ikeda's method proved to be the most precise in the present study even with sample sizes as low as 10. Ramsay's and Dunnet's R_t/ϕ plots needed 25–50 clasts to retain a precision of near 10%. Elliott's polar plots needed 50–90 markers. The least precise method proved to be the X/Y plots.

CONCLUSIONS

The foregoing discussion points out some of the potential hazards encountered in strain analyses. The following procedure represents the most precise method determined in this study for analysing strain data from ellipsoidal objects. The method provided strain data with a precision of $\pm 5\%$ and maximized the amount of information obtained for the hours invested.

Oriented samples, each approximately 10-20 cubic centimeters, containing as many ellipsoidal objects as possible are located. A complete survey that includes the orientations of foliations. lineations, and bedding, local variations in strain (calculated in the field using the harmonic mean) and lithologies, the position of the sample within larger structural elements, and evidence for the non-homogeneity of strain mechanisms is completed for each sample. The sample is then taken to the laboratory where three orthogonally cut slabs are removed, coated with acrylic spray, and reoriented from field data. Thin sections are made from the slabs and analysed for textures and mineralogy. The elliptical objects in each slab are measured using a monocomparater with the data being fed directly into a computer filespace. The data are separated according to the type of object, lithology and size (measured in terms of area or long axis of the ellipses), as each of these factors affect the amount of strain recorded.

Once the data have been transferred into the filespace, a computer program calculates two-dimensional strain

ratios using Shimamoto & Ikeda's (1976) algebraic and Lisle's (1977) theta-curve method. The same program is designed to calculate strain ratios and additional information using any of the methods discussed in this paper by calling up subroutines representing the particular method. Three dimensional data are obtained from Miller & Oertel's (1979) method (or PHASE 5 in the future). Depending on the types of strains occurring, Gay's (1968) method may also be used. As new methods become available they may in turn simply be added to the program.

Acknowledgements—I would like to thank Margaret Whalen, Steve Johnston and Laurel Goodwin for their constant support during the completion of this project. Drs. Philip H. Osberg, Stephen A. Norton and Terence J. Hughes provided many helpful suggestions. Many of the authors cited in this paper provided programs, suggestions and encouragement. The project was financed by a grant from Sigma Xi and the Department of Geology, University of Maine at Orono.

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